Digital Signals

Assignment 3: Due 7/22/2016

Nicholas Dwork

1 Main Problems

Problem *Alpha-blending* If you are given two points (or vectors) p_1, p_2, then you can parameterize the line segment between them as follows:

$$f(\alpha) = \alpha\, p_1 + (1-\alpha)\, p_2,$$

where $\alpha \in [0,1]$. What value does the function take on when $\alpha = 0$, $\alpha = 0.5$, $\alpha = 1$.

We can think of an image as a point and use the above parameterization to make a line segment between two images (gnarly, I know). In this problem we are going to visualize this line. Download two images from the Internet that you like. Change these images so that they are the same size. Make an array of α values between 0 and 1 in steps of 0.2. Compute $f(\alpha)$ for each value of α where p_1 is one image and p_2 is the other. Save each image and include them in your final document.

Problem Consider the optimization problem below

$$\text{minimize } \|Ax-b\|_2^2 + \gamma \|Dx\|_2^2$$

where A, b, γ, D are known, and x is the optimization variable. Find matrix $\tilde{A}$ and $\tilde{b}$ such that the following optimization problem is equivalent to the one above.

$$\text{minimize } \|\tilde{A}x - \tilde{b}\|_2$$

Provide an analytic expression for the optimal value of x in terms of $\tilde{A}$ and $\tilde{b}$.

Problem The video at the following link is of a real helicopter flying around; there are no special effects in the video.

`https://www.youtube.com/watch?v=qgvuQGY946g`

Why does it look like the blades aren't spinning?

Problem Recall that for a very long time, philosophers held a widespread agreement that the definition of knowledge was "a justified true belief." Create your own Gettier case that violates this definition. Can you create a better definition of knowledge?

Problem Find a subset of $\mathbb{R}^2$ that satisfies each of the following:

- Closed under pointwise addition but not under scalar multiplication
- Closed under scalar multiplication but not under pointwise addition

Problem *Multiple Linear Regression* With multiple linear regression, you are given a set of inputs and an output. The output is dependent on each input according to the following affine model:

$$f(x_1, x_2, \ldots, x_N) = \alpha_1 x_1 + \alpha_2 x_2 + \cdots + \alpha_N x_N + \epsilon.$$

where f outputs a scalar, and α_i is a scalar for all i, and ϵ is a scalar. Consider the case where you are given many different sets of inputs and corresponding outputs. The goal is to determine the coefficients $\alpha_1, \alpha_2, \ldots, \alpha_N$, and ϵ. Show that there exists a matrix A and a vector b such that $A\gamma = b$, where $\gamma = (\alpha_1, \alpha_2, \ldots, \alpha_N, \epsilon)$.

Problem Read some part of Reality is Broken. Describe how it adheres to and/or conflicts with conventional thought. What is your opinion of what you've read? (Please state which pages you read.)

Problem You are given N pairs of points (x_i, y_i) that adheres to the following relationship: $y_i = f(x_i) + n_i$ where f is a fourth degree polynomial and n is a small amount of noise. Your goal is to determine the function f. Download the python code at

`www.stanford.edu/~ndwork/si2016/session2/hmwk3/polyFit.py.`

In your answer provide the coefficients of the polynomial and plot your function f on the same graph as the data points.

Problem I have taken a video of imagery from a stationary platform. You can download individual frames from this video here:

`www.stanford.edu/~ndwork/si2016/session2/hmwk3/peopleWalking.zip`

The goal of this project is to create a video of only the background (without any people in it). Note that there is no single frame like this; we're going to have to do some image processing to achieve our goal.

Part a Work completely in grayscale (that is, work with 2D images rather than 3D images; one way to convert from 3D to 2D is to only use one color channel). Think of a grayscale video as a 3D array, where each slice of the array is a single image. Make a new image where the value of each pixel equals the temporal mean of that pixel in all frames. That is, if the pixel we are computing is the $(i, j)^{\text{th}}$ pixel, the value of that pixel is the median of the $(i, j)^{\text{th}}$ pixel in all frames.

Part b Repeat step (a), but this time work with color imagery.

Part c Why does this work?
Please include before and after pictures in your submission.

Problem *Message Obfuscation* In class we learned one way to obfuscate a message by hiding it in an image; this project will show you another way to do that. As we previously discussed, each pixel of an image is a number that ranges from 0 to 255. This is because we store the data in a pixel as a byte, which is an ordered set of eight bits (a bit is a single digit that can store either 0 or 1). The conversion from binary to decimal is as follows:

$$x_7\, x_6\, x_5\, x_4\, x_3\, x_2\, x_1\, x_0 = 2^7 x_7 + 2^6 x_6 + \cdots + 2^0 x_0.$$

For example:

$$10110010 = 2^7 + 2^5 + 2^4 + 2^1 = 178.$$

The quantity x_0 is called the *Least Significant Bit* or LSB. Note that the gray of 178 is extremely close to the gray of 179. So if we added or subtracted 1 to the value it wouldn't change the image much.

To encode an ASCII message into an image, subtract 1 from all odd values. What is the LSB after we do this? Then, add the message bit (1 or 0) to each pixel in sequence until the message is completed. Do not add anything to the remaining pixels.

To convert from ASCII to binary (and vice-versa) you can use the following website:

`http://www.unit-conversion.info/texttools/convert-text-to-binary/#data`

Extract the message in the image located at

`www.stanford.edu/~ndwork/si2016/session2/hmwk3/bacEmbedded.png.`

In this image, the ASCII code was embedded in the image in column-major order.