# Fundamentals

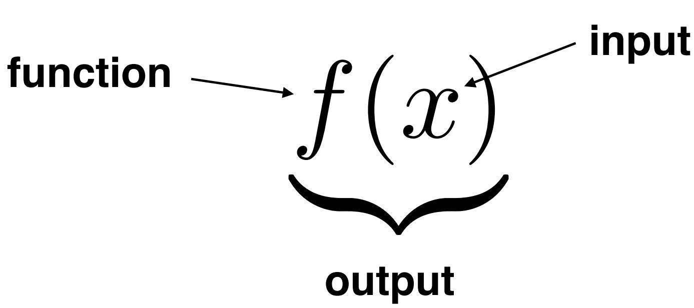
## Math Lecture 1

**Nicholas Dwork**



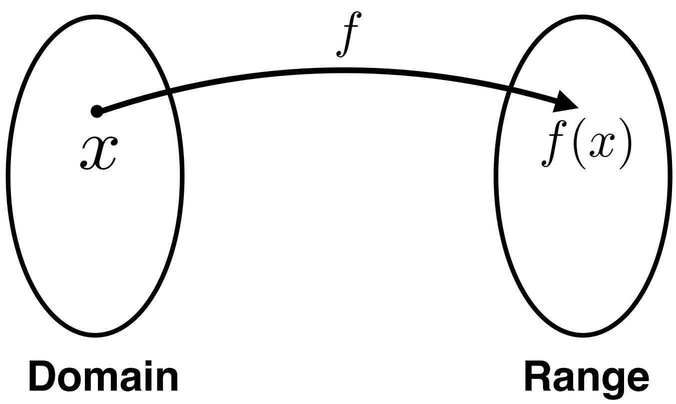
# Functions

**A function is a mathematical machine**
**You input something**
**You get something out**




**As long as you input the same thing, you'll always get the same thing out.**

Consider a function $f$ . What is the precise definition of a function?



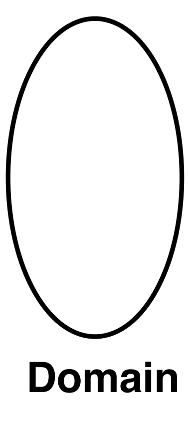

As the picture above indicates, a function has three parts. We define a function as an ordered triple.



# Domain

A function is an ordered triple.

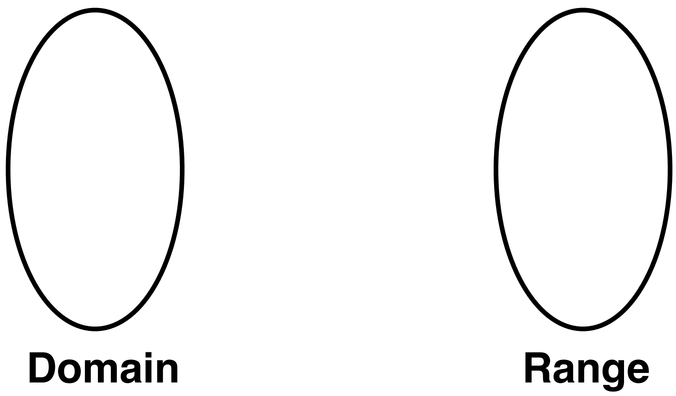
The first element is a set called the Domain.



# Range

**A function is an ordered triple.**

**The second element is a set called the Range.**

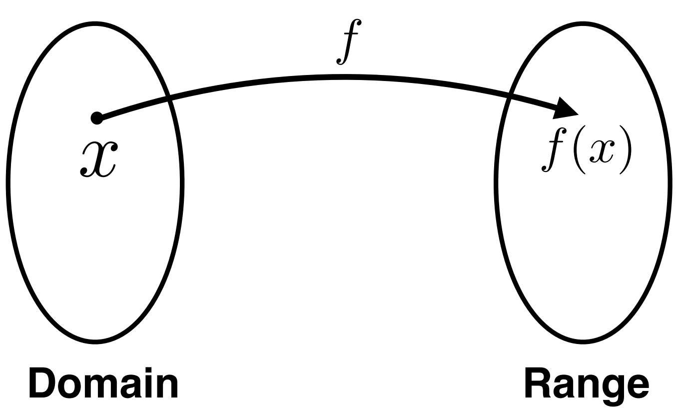





# Set of Ordered Pairs

**A function is an ordered triple.**

**The third element is a set of ordered pairs that matches elements in the domain with elements in the Range.**



# Set of Ordered Pairs

**For each element in the Domain there is an ordered pair.**

**The first element in the pair is the Domain element. This is often called the "input".**

**The second element of the ordered pair is an element of the Range. Its often called the "output" for that input.**

$$(x, f(x))$$



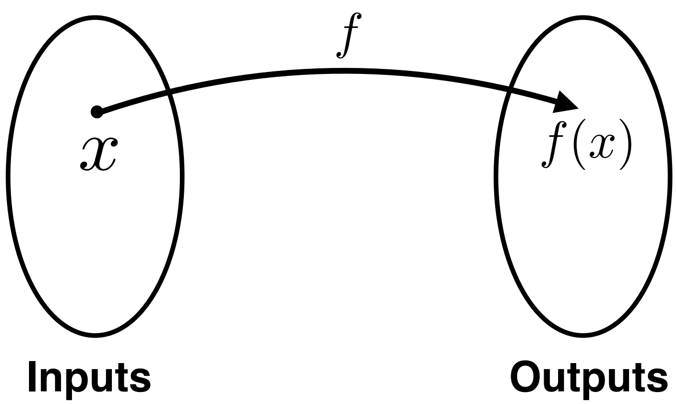
# A Function

**Domain** **Range**

$$f = (D, R, S)$$

$$S = \{(x, f(x)) : x \in D\}$$

**A function maps inputs to outputs. It converts $x$ to $f(x)$.**






# Function Notation

$$f : D \to R$$

**This is followed by "such that" and then a rule specifying $f(x)$.**

# Functions - Example 1

$f : \mathbb{R} \rightarrow \mathbb{R}$ **such that** $f(x) = x$



# Functions - Example 2

**Domain** **Range**

**( {a,b,c,d}, {0,1,2,3,4,5,6,7,8},**
**{ (a,0), (b,0), (c,8), (d,6) } )**

**S**

**This is a completely valid function!**

# Functions - Example 3

$f : \mathbb{R} \to 0, 1$ **such that**

$$f(x) = \begin{cases} 0 & \text{if } x \text{ is irrational} \\ 1 & \text{if } x \text{ is rational} \end{cases}$$



# Example Function

**The exponential function:** $\exp$

$$\exp(-1) = 0.3679$$

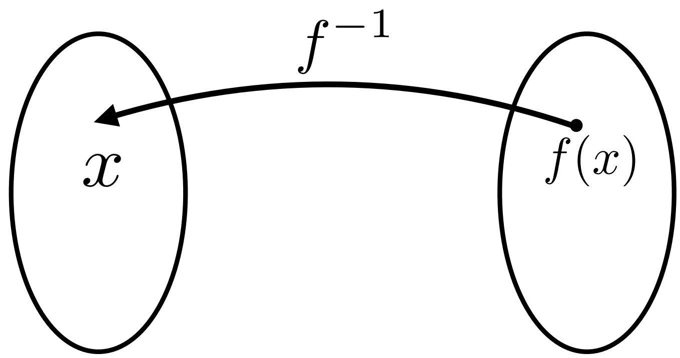
$$\exp(0) = 1$$

$$\exp(1) = 2.7183$$

$$\exp(1.2) = 3.3201$$

# Inverse Function

**The inverse function converts all $f(x)$ in Outputs back to $x$ in Inputs.**




**Note: Not every function has an inverse function. An invertible function is a very special thing.**



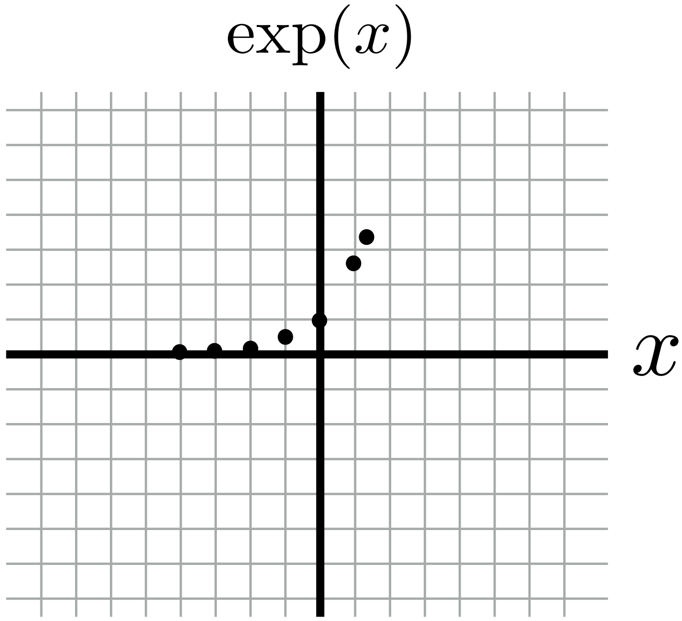
# Example Inverse Function

**The inverse of the $\exp$ function is the $\log$ function.**

| | |
|---|---|
| $\exp(-1) = 0.3679$ | $\log(0.3679) = -1$ |
| $\exp(0) = 1$ | $\log(1) = 0$ |
| $\exp(1) = 2.7183$ | $\log(2.7183) = 1$ |
| $\exp(1.2) = 3.3201$ | $\log(3.3201) = 1.2$ |

# Graphing a Function

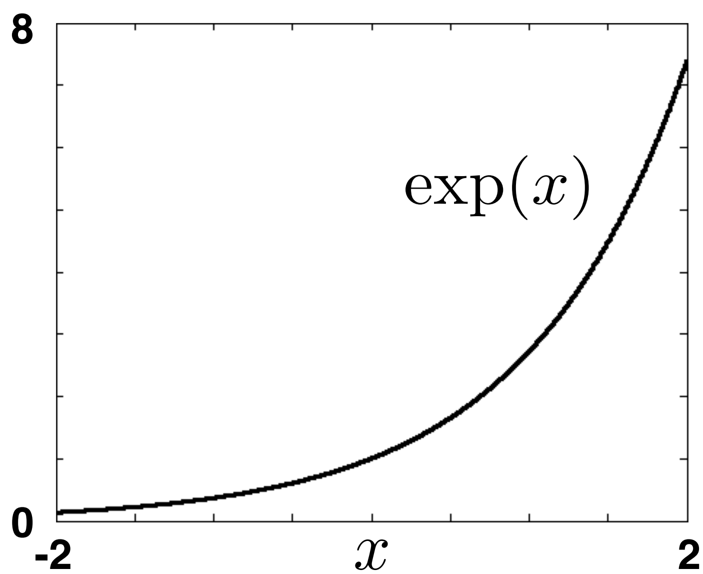
**Showing the points of a function in a Euclidean Plane**





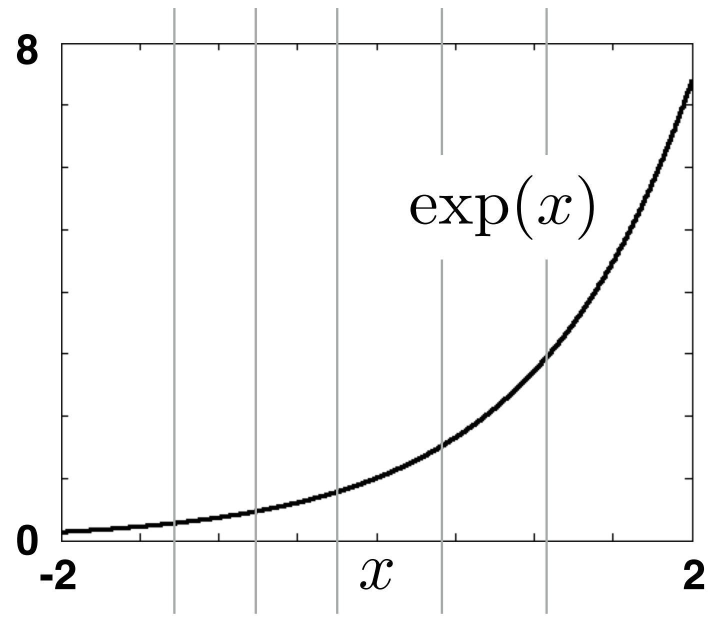

# Graphing a Function

**Showing all the points of a function in a Euclidean Plane**



# Vertical Line Test

**If you draw a vertical line through the graph of a function, it will intersect at most one point.**

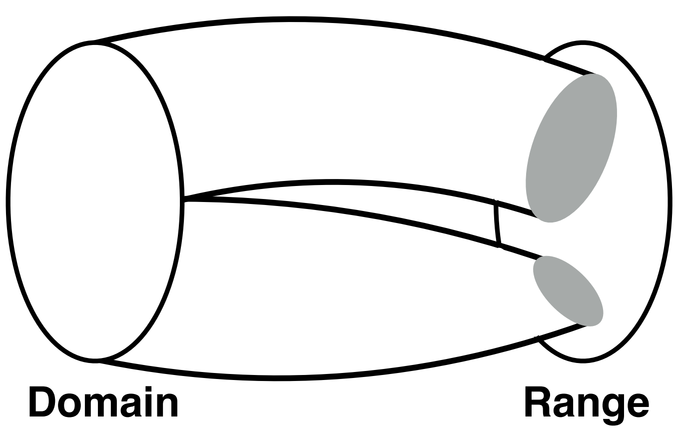





# Image

**The set of all attainable outputs is called the Image.**

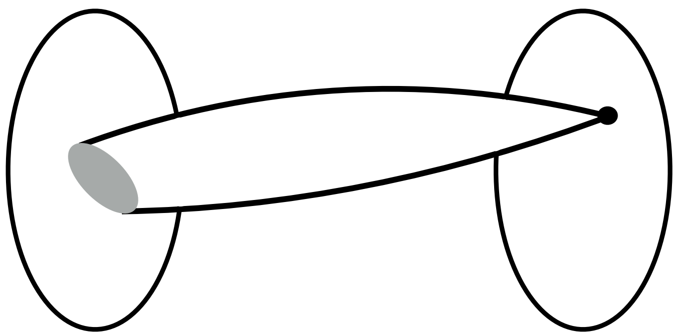
**The Image of a function is a subset of the Range.**



# Pre-image

**The set of all inputs that map to a value is called the pre-image of that value for the function.**

**The Pre-image of a value is a subset of the Domain.**



**Domain** **Range**



# Equation of a Line

**One way to represent a line is with a function of the form**

$$f(x) = m\,x + b$$

*m* **is called the "slope" of the line.**
*b* **is called the "vertical intercept" of the line.**

**We can't represent vertical lines this way. We'll see a more general representation later.**