Introduction to Computer Graphics and Computer Vision

Assignment 4: Due 7/30/2015

Nicholas Dwork

1 Main Puzzles

P 1 In class we showed the equation of a sinusoid that represents a wave as follows:

$$f(x,t) = A\sin(kx + \omega t + \phi).$$

Note: unless otherwise specified, the value of the parameter is 0.

Part a Make a plot of f where x ranges from 0 to 10 meters. k be 1 cycle per meter (recall that the input to sin is in radians and not cycles, so you'll have to make the appropriate conversion). When making this plot, set the linewidth to 2 (it makes the line thicker on the plot; ask the TAs for help if you need it). What happens when you change k to 1.5? Plot this on the same plot with a different color for the new plot.

Part b The value ϕ is called the "initial phase". Try changing phi to 0.1; what happened? Plot this on the same plot with a different color for the new plot. Use the `legend` command to label each plot. Submit this plot and explain in words what happened. What do k, ϕ do?

Part c Now let ω be 1.5 radians per second, and let t vary from 0 to 10 seconds in steps of 0.1 seconds. At each time point, make a plot of the function (use the same figure, so the plot overwrites itself). You should see an animation of how the wave changes with time. Make a movie of this animation using Matlab's `movie2avi` function (you can see an example of how this is done here: http://stackoverflow.com/questions/11051307/approaches-to-create-a-video-in-matlab). Upload this video to your Google Drive or Dropbox account and provide a link to the video in your solutions.

Part d So far, we have been talking about waves in 1D; now we're going to look at waves in two dimensions. The two dimensional equation for a wave is

$$f(x,y,t) = A\sin(k_x x + k_y y + \omega t + \phi).$$

The value k_x is called the x spatial frequency, and k_y is called the y spatial frequency. Let t and ϕ be 0. Make plots of the following pairs for k_x and k_y: $(1.5, 0)$, $(1.5, 1.5)$, $(0, 1.5)$, $(-1.5, 1.5)$. What changes?

Part e For $(k_x, k_y) = (1.5, 1.5)$, let t vary from 0 to 10 seconds in steps of 0.1 second with $\omega = 1.5$ radians per second. Keep $\phi = 0$. At each time, plot the image using `imshow`; make an avi movie of this animation and upload it to your Google Drive or Dropbox account; provide a link to the video in your solution.

Part f In your own words, explain what k_x, k_y, ω, and ϕ are. What intuition have you developed in this problem?

P 2 (*Credit: Dan Meyer*) Download the video at the following website. Rank all the drinks from strongest caffeine concentration to weakest caffeine concentration. Which drink gives you the biggest *bang* for your buck?

`www.stanford.edu/~ndwork/si2015/hmwk3/act1.mov`

(Like before, you need information you don't have. Figure out what information you need and ask the TAs for it.)

P 3 Download the zip file from the following website. If you extract the contents of the zip file, you'll get a directory with a bunch of images inside. Each image is a frame of a video; it is of a courtyard with some people moving around it.

`www.stanford.edu/~ndwork/si2015/imgs/peopleWalking.zip`

Part a Take the temporal mean of the images (first converted to grayscale using `rgb2gray`). That is, for each pixel, compute the average value of that pixel over the time of the entire video. Show the "temporal mean" image. What happened? Can you explain why?

Part b Take the temporal median of the images (first converted to grayscale using `rgb2gray`). That is, for each pixel, compute the median value of that pixel over the time of the entire video. Show the "temporal median" image. What happened? Can you explain why?

Part c Make a video where each frame of the video is the original frame minus the temporal median frame. What is the result? Can you explain why?

Part d Do it all again, this time in color.

P 4 Due to the imperfect nature of a lens, an image will suffer from some amount of geometric distortion. Due to the spherical nature of a lens, Radial distortion is often a good model for the geometric distortion. Radial distortion is modeled as follows

$$\begin{bmatrix} \tilde{x} \\ \tilde{y} \end{bmatrix} = L(r) \begin{bmatrix} x \\ y \end{bmatrix},$$

where (x, y) is the point in the image that a ray would project to without any distortion, $(\tilde{x}, \tilde{y})$ is the point the ray projects to with distortion, r is the distance between (x, y) and the principal point. L can be modeled using the following formula

$$1 + \kappa_1 r + \kappa_2 r^2 + \kappa_3 r^3 + \cdots$$

For the purposes of this puzzle, we will restrict L to be a fourth order polynomial. Let $\kappa_1 = 1.03689, \kappa_2 = 0.0487908, \kappa_3 = 0.0116894, \kappa_4 = 0.00841614$. Use these values to apply radial distortion to the image downloaded from the following website address. For these values to work, r must be normalized so that the largest radius is 1.

`www.stanford.edu/~ndwork/si2015/imgs/bac.jpg`

P 5 Also due to the imperfect nature of a lens, an image will suffer from some amount of light attenuation as a function of angular distance from the principal ray. This phenomenon is called *Vignetting*. An example of such an image is shown in the figure below.

Figure 1: This image shows an example of vignetting. The image gets darker closer to the boundaries.

Part a Vignetting can be modeled with a $\cos^4(\theta)$ where θ is the angular displacement of the pixel away from the principal ray. Without knowing the intrinsic parameters of the camera that took the image, we can't know θ, but we can use reasonable parameters to simulate vignetting. See if you can apply vignetting to the image downloaded from the following address. (We can use this procedure to simulate vignetting in computer generated images.)

`www.stanford.edu/~ndwork/si2015/imgs/bac.jpg`

Part b See if you can remove the vignetting effect from the image of Figure 1. You can acquire that image at the following address.

`www.stanford.edu/~ndwork/si2015/imgs/vignetting.jpg`

P 6 We can use anaglyph (red-blue) glasses to experience stereo perception. For this puzzle, download the images `left.jpg` and `right.jpg` from the following website.

`http://stanford.edu/~ndwork/si2015/imgs/anaglyph/`

Convert these images to grayscale. Then apply a Homography to project left onto right. Finally, use these images to make a color image where the red channel is comprised of the projected left image, and the blue channel is comprised of the right image. You should be able to view this color image with anaglyph glasses and experience stereo perception.

2 Challenge Puzzles

CP 1 Prove the following statement:

$$\sum_{k=0}^{n} a^k = \frac{1 - a^{n+1}}{1 - a}.$$

CP 2 Prove that there are infinitely many primes. (Again, this is a proof by contradiction. Feel free to use all the statements of real numbers and integers that you know to be true to solve this one.)