Introduction to Computer Graphics and Computer Vision

Assignment 1: Due 7/20/2015

Nicholas Dwork

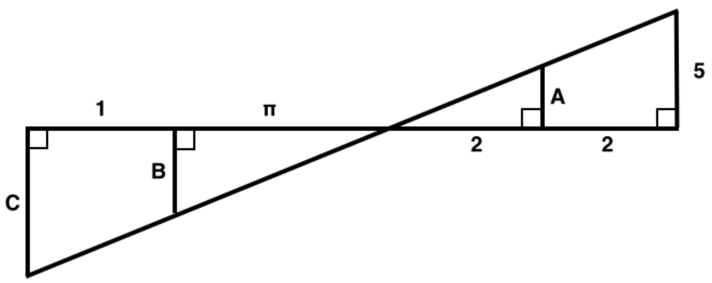
1 Main Problems

Problem 1 Visit https://studio.code.org/hoc/1 and complete the first 20 puzzles.

Problem 2 Explain what the following lines of Matlab code do:

```
v = 'v';
hiThere = 'Hi There';
asdf = 88;
eight = 8;
```

Problem 3 Find the lengths of lines $A, B,$ and C in the image below.



Problem 4 Explain how the equation of a circle follows from the Pythagorean theorem.

Problem 5 Show in three dimensions that the length of a vector a is $\|a\|_2$. (Draw a picture as part of your explanation.)

Problem 6 (*Credit: Dan Meyer*) For this problem you will need to view videos on the class website.

Part a At the following website, you will see a video of a jug getting filled with water. How long will it take to fill up the jug? (You, of course, realize that you will need information to solve this problem. Identify the information you think you need and then ask the TAs for it. They have all the relevant information.)

```
www.stanford.edu/~ndwork/si2015/hmwk1/act1.mov
```

Once you have an answer, present it to a TA. He will show you another video. Why is your answer different from the video?

Part b At the following website, you will see a video of a jug getting getting emptied. How long will it take the jug to empty? (Again, ask the TAs for the information you need.)

```
www.stanford.edu/~ndwork/si2015/hmwk1/act1-sequel.mov
```

Once you have an answer, present it to a TA. He will show you another video. Why is your answer different from the video?

Problem 7 (*Credit: Dr. Boyd*) Let x be a block vector with two vector elements,

$$x = \begin{bmatrix} a \\ b \end{bmatrix},$$

where a and b are vectors of size m and n, respectively.

Part a Express mean(x) in terms of mean(a) and mean(b).

Part b Express $\|x\|_2$ in terms of $\|a\|_2$ and $\|b\|_2$.

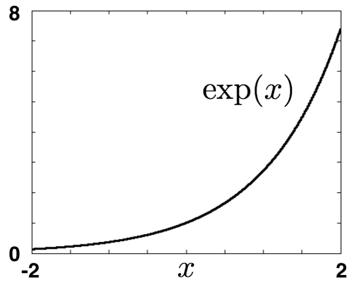
Problem 8 (*Credit: Dr. Boyd*) Show that $\text{mean}\,(\alpha\, x + \beta 1) = \alpha\,\text{mean}(x) + \beta$.

Problem 9 It's easy to tell whether or not a function is invertible on a finite domain given its graph. How can you tell?

Problem 10 Write a function that accepts three (x, y) points and returns the area of a triangle. The prototype of the function should be as follows:

```
function area = areaOfTriangle( x1, y1, x2, y2, x3, y3 )
```

Problem 11 Graph the inverse function of exp for all the points shown on the following graph.



Problem 12 Find the inverse of the following matrix without using a computer.

$$\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$$

Problem 13 Write a function that accepts a natural number and outputs a triangle of numbers so that the elements in each column indicate the reverse column id. The prototype of the function should be as follows:

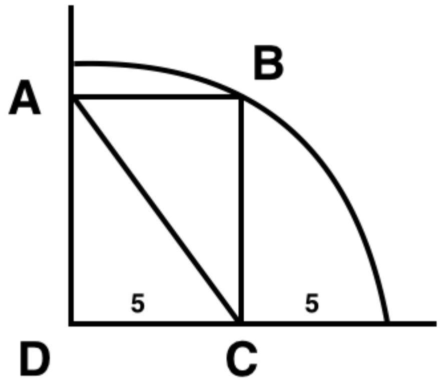
```
function makeNumberTriangle( N )
```

A sample output where the input number $N = 4$ is shown below

$$\begin{matrix} & & & 1 \\ & & 2 & 1 \\ & 3 & 2 & 1 \\ 4 & 3 & 2 & 1 \end{matrix}$$

Make sure the function dies nicely if someone inputs something other than a natural number (look at the error function in Matlab).

Problem 14 Find the length of line AC. (Hint, this should take you almost no time at all). *Source: My Best Mathematical and Logic Puzzles by Martin Gardner.*



Problem 15 *Properties of the cross product.*

Part a Prove that $a \times b$ is perpendicular to both a and b.

Part b Prove that the cross product of parallel vectors is 0.

Part c How is $a \times b$ related to $b \times a$?

Part d Given a vector a, find a matrix M_a such that $M_a b = a \times b$ for all vectors b.

Problem 16 Create a matrix of zeros 500 × 500 in size called "myImage". Then execute the following command:

```
myImage(1:100,1:200) = 1;
```

Display this image on the screen. What does this show you about image coordinates in Matlab? (Provide as many conclusions as you can.)

Problem 17 *Commutativity of Matrices*

Part a Either prove that matrices commute with addition or give a counter example.

Part b Either prove that matrices commute with multiplication or give a counter example.

2 Challenge Problems

CP 1 Make a function that accepts a natural number as input and outputs a boolean. The function must determine whether or not the input problem is prime; it returns True if the number is prime and False otherwise. The function prototype is as follows.

```
function out = isPrime( N )
```

CP 2 A function f is continuous at a point x_0 means $\lim_{x \to x_0} f(x) = f(x_0)$. A function f is continuous means that it is continuous at every point in its domain. Pointwise addition for functions means $(f_1 + f_2)(x) = f_1(x) + f_2(x)$. Scalar multiplication for functions means $(\alpha f)(x) = \alpha (f(x))$. Prove that the set of continuous functions whose domain is $\mathbb{R}$ together with pointwise addition and scalar multiplication is a vector space.

CP 3 Let a and b be the following vectors:

$$a = \begin{bmatrix} 4 \\ 1 \end{bmatrix} \quad b = \begin{bmatrix} 3 \\ 8 \end{bmatrix}.$$

The vector b can be written as the sum of two vectors, one that's parallel to a and one that's perpendicular to a. Find these vectors. (Note: The vector that is parallel to b is called "the projection of a onto b".)