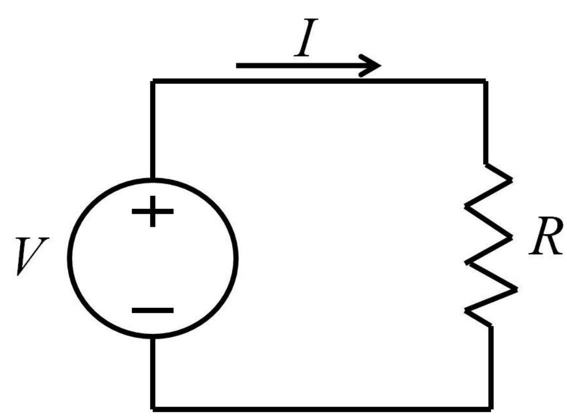
Signal Processing and Linear Systems 1
Circuits Review

Units and Ohm's Law

$$V=IR$$

- Units
  - Voltage (V): volts (V) (= J/C)
  - Current (I): amps (A) (=C/s)
  - Resistance (R): ohms (Ω)



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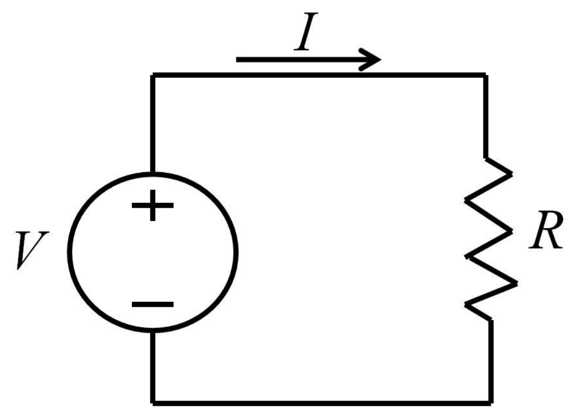
- Water Analogy
  - Voltage ~ water pressure
  - Current ~ flow rate
  - Resistance ~ pipe width



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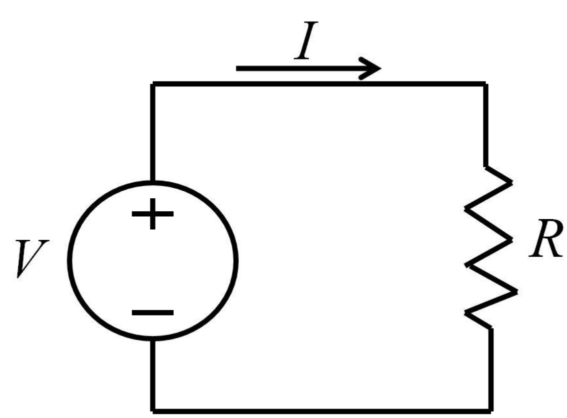
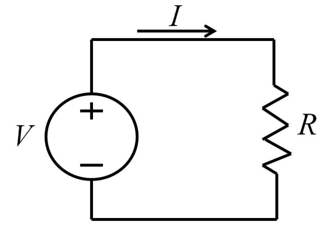
- Conventional Current
  - Current flows from high to low voltage by convention

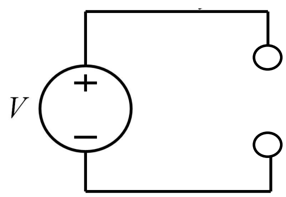


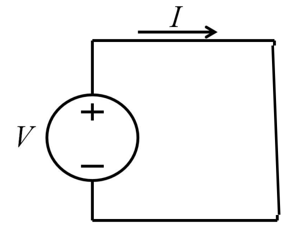
Open and short



V=IR

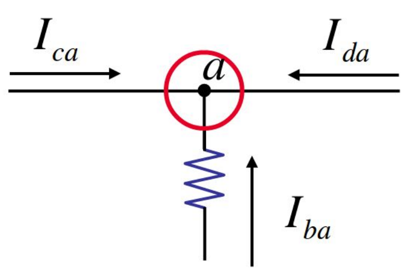
- When R = ∞, we have an open circuit
- When R = 0, we have a short





Kirchoff's Current Law (KCL)

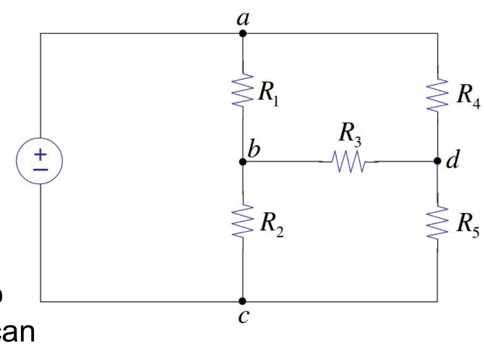
$$\sum_{k=1}^{n} I_k = 0$$



$$I_{ca} + I_{da} + I_{ba} = 0$$

- The sum of current into any node is zero
- Current flowing towards a node given positive sign, while current flowing away given negative sign
- This Law follows from the conservation of charge
- Water analogy: net water flowing into any point must be zero

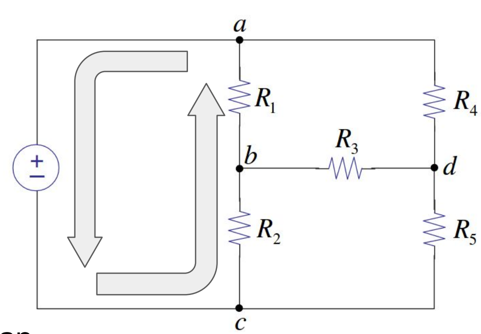
Kirchoff's Voltage Law (KVL)



$$\sum_{k=1}^{n} V_k = 0$$

- The sum of voltages along and loop in zero
- Each voltage as you move along the loop can be positive or negative
- This Law follows from the conservation of energy

Kirchoff's Voltage Law (KVL)



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$v_{ac}+v_{cb}+v_{ba}=0$

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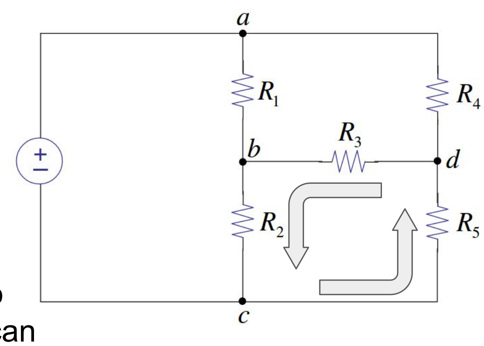

$v_{ac}+v_{cb}+v_{ba}=0$

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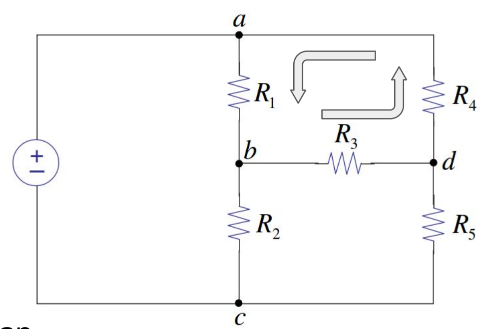


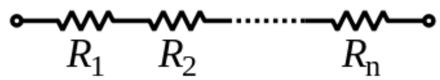
$v_{ac}+v_{cb}+v_{ba}=0$

$v_{bc}+v_{cd}+v_{db}=0$

$v_{ab}+v_{bd}+v_{da}=0$

Resistors in Series

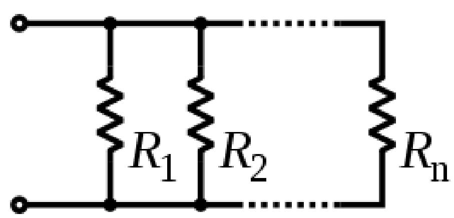
$$R_{\text{eq}} = R_1 + R_2 + \cdots + R_n$$

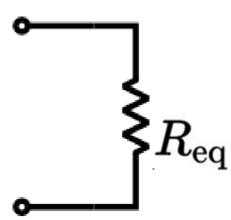


- The equivalent resistance of resistors connected in series is the sum of the individual resistances
- This rule follows from Ohm's law and KCL
  - A consequence of KCL is that portions of circuit that are in series must have the same current. Here, the current through each resistor must be equal.

Resistors in Parallel

$$\frac{1}{R_{\text{eq}}} = \frac{1}{R_1} + \frac{1}{R_2} + \cdots + \frac{1}{R_n}$$



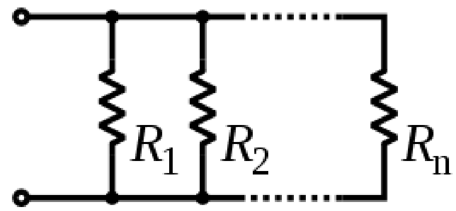


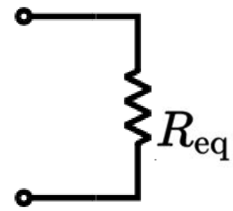
- The equivalent resistance of resistors connected in parallel is the harmonic mean of the individual resistances
- This rule follows from Ohm's law and KVL
  - A consequence of KVL is that portions of circuit that are in parallel must have the same voltage. Here, the voltage across each resistor must be equal.

Resistors in Parallel - Water Analogy

$$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + \cdots + \frac{1}{R_n}$$

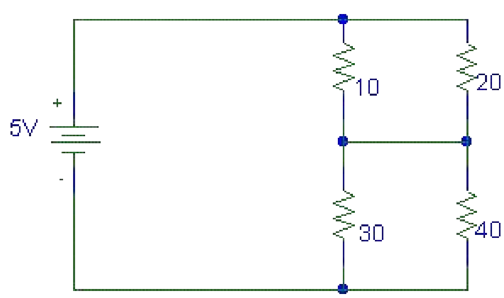
- Imagine two water pipes in series, when very skinny (high resistance), one very wide (low resistance).
- The overall resistance will be determined by the wider pipe
- Most of the water (current) will flow through the wider pipe





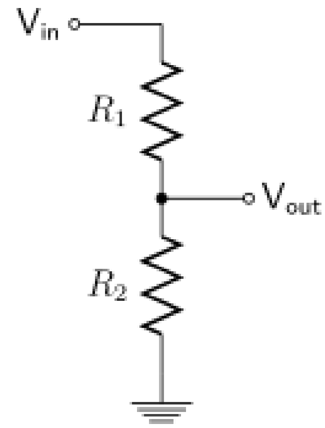
Example of Combining these Rules

- What is the equivalent resistance of the four resistors?
- What is the current flow through each resistor?
- (work out on board)



Voltage Divider

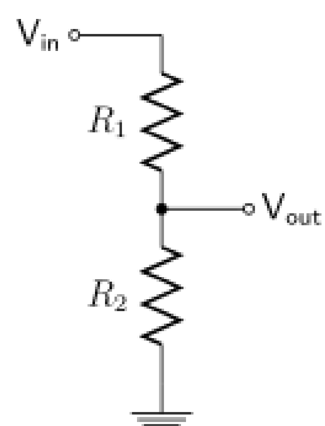
- What is V_{out}? As function of V_{in}?
- (work out on board)



Voltage Divider

- What is V_{out}? As function of V_{in}?
- Combined resistance is R_1+R_2
- Thus, current is $I=V_{in}/(R_1+R_2)$
- $V_{out}=I*R_2=(V_{in}/(R_1+R_2))*R_2$
- $V_{out}/V_{in}=R_2/(R_1+R_2)$

$$V_{\text{out}} = \frac{R_2}{R_1 + R_2} \cdot V_{\text{in}}$$



Resistors, Capacitors, and Inductors

Resistor

$$V = IR$$

- Physically, a resistor is usually a strip of partially insulative material

Capacitor

$$I = C\frac{dV}{dt}$$

- Physically, a capacitor is usually two parallel charged plates seperated by an insulator

Voltage

$$V = L\frac{dI}{dt}$$

- Physically, an inductor is usually a coil of wire
- It opposes change in current

Impedances

- Capacitors and Inductors have a complex valued "resistance"

$$Z_C = \frac{1}{j\omega C}$$

Capacitor

$$Z_L = j\omega L$$

Inductor

$$Z_R = R$$

Resistor

where ω is the frequency

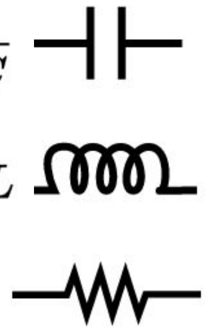
Impedances

- Capacitors and Inductors have a complex valued “resistance”
- This generalized notion of resistance is known as impedance.

$Z_C = \frac{1}{j\omega C}$ Capacitor

$Z_L = j\omega L$ Inductor

$Z_R = R$ Resistor



where ω is the frequency

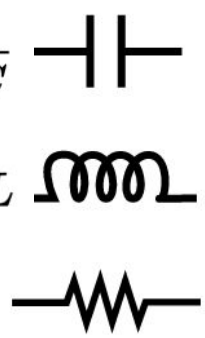
Impedances

- Capacitors and Inductors have a complex valued “resistance”
- This generalized notion of resistance is known as impedance.
- Note limiting behavior
  - At high frequency capacitor become shorts and inductors opens
  - At low frequency capacitors become opens and inductors shorts

$Z_C = \frac{1}{j\omega C}$ Capacitor

$Z_L = j\omega L$ Inductor

$Z_R = R$ Resistor



where ω is the frequency

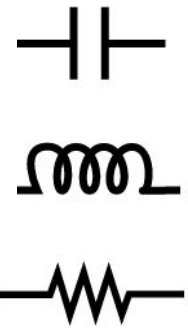
Impedances(cont'd)

- When resistors, capacitors, and inductors are combined in circuits their impedances are combined by same rules we saw for resistors.
- We will look at a example on the next slide

$$Z_C = \frac{1}{j\omega C}$$

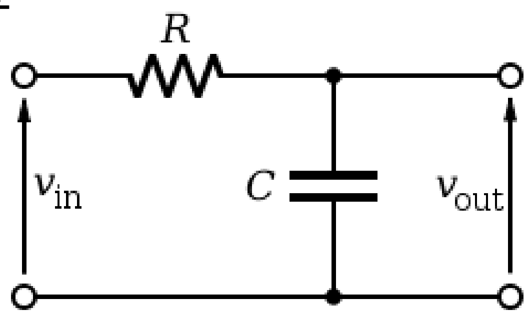
$$Z_L = j\omega L$$

$$Z_R = R$$

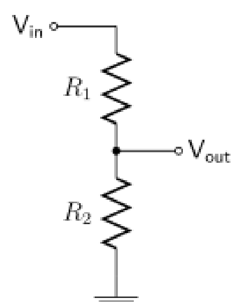


RC Low-Pass Filter

- This RC circuit is the same as the voltage divider we saw earlier except that we are now replace resistances with impedances.
- V_{out}/V_{in} is known as the transfer function.



Low-Pass Filter



Voltage Divider

$$\frac{V_{out}}{V_{in}} = \frac{\frac{1}{j\omega C}}{R + \frac{1}{j\omega C}} = \frac{1}{1 + j\omega RC}$$

$$V_{\text{out}} = \frac{R_2}{R_1 + R_2} \cdot V_{\text{in}}$$

RC Low-Pass Filter (cont'd)

- We can examine both the magnitude and the phase of this complex valued transfer function.

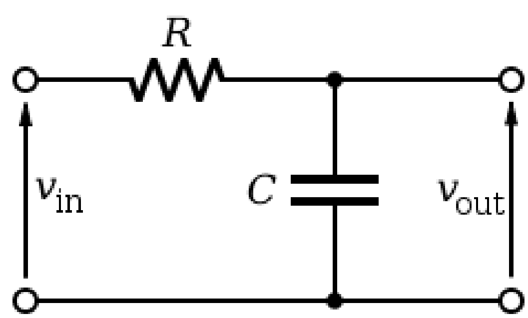
$$\frac{V_{out}}{V_{in}} = \frac{1}{1 + j\omega RC} = \frac{1}{1 + j\frac{\omega}{f_c}}$$

$$|\frac{V_{out}}{V_{in}}| = \frac{1}{\sqrt{1 + (\frac{\omega}{f_c})^2}} \qquad f_c = \frac{1}{RC}$$

$$\angle(\frac{V_{out}}{V_{in}}) = -\angle(1 + j\frac{\omega}{f_c}) = -\arctan(\frac{\omega}{f_c})$$

RC Low-Pass Filter (Cont'd)

- Recall that at low frequency a capacitor is like an open and at high frequency it is like a short. Let's check if this matches low pass behavior
- Indeed, if the capacitor is replaced with an open, $V_{out} = V_{in}$
- If the capacitor is replaced with a short, $V_{out} = 0$.
- This if often a good check to do.
- Exercise: What sort of filter will we get if we switch the place of the resistor and the capacitor? What will be transfer function be?



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