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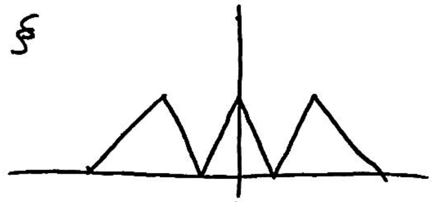
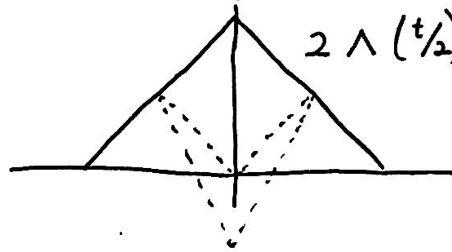
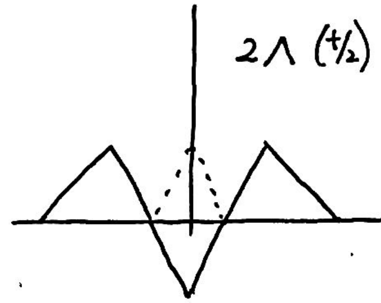


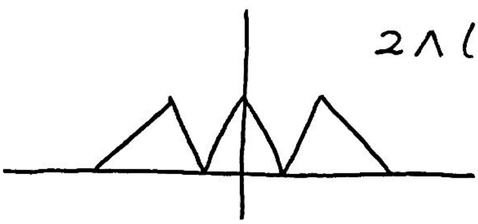
$2\Lambda(t/2)$



$2\Lambda(t/2) - (2\Lambda(t) + 1)$



$2\Lambda(t/2) - (2\Lambda(t) + 1) + 2\Lambda(2t)$



§ System Review

Let's think of a function: $f(x) = x^2$.
f is the name of the function.
We can put a number into this function and find its corresponding output.
Ex: $f(0) = 0$. $f(2) = 4$.

A system accepts a function as input; its output is another function.
$S\{z\}$ means we have sent the function z into the system. And the output is the function named $S\{z\}$.

So $S\{z\}$ is like 'f' in our earlier example. We can send a number into this function and see its corresponding output.

Ex: it may be the case that $S\{z\}(x) = x^2$.
So we can find outputs.
$S\{z\}(0) = 0$. $S\{z\}(2) = 4$.

What's an example of a system?

$$S\{z\}(\gamma) = \int_{-\infty}^{\gamma/2} z(\tau)\,d\tau.$$

So, for example, we can see what happens when we send $z(t) = e^t$ into our system.

$$S\{z\}(\gamma) = \int_{-\infty}^{\gamma/2} e^{\tau}\,d\tau = e^{\tau}\Big|_{-\infty}^{\gamma/2} = e^{\gamma/2}.$$

So $S\{z\}(\gamma) = e^{\gamma/2}$.

What if $z(t) = t\,u(t)$?

$$S\{z\}(\gamma) = \int_{-\infty}^{\gamma/2} \tau\,u(\tau)\,d\tau = \int_{0}^{\gamma/2} \tau\,d\tau = \gamma^2/8.$$

So $S\{z\}(\gamma) = 1/8\,\gamma^2$.

How can we tell if this system is shift invariant?
We need to see what happens when we send in a shifted function.

Ex: $S\{g(t)\}(\gamma) = g(\gamma)\cos(\gamma+\phi)$.

Let $f(t) = g(t-\Delta)$.

$$\begin{aligned} S\{f(t)\}(\gamma) &= f(\gamma)\cos(\gamma+\phi) \\ &= g(\gamma-\Delta)\cos(\gamma+\phi). \end{aligned}$$

$$S\{g(t)\}(\gamma-\Delta) = g(\gamma-\Delta)\cos(\gamma-\Delta+\phi).$$

$\therefore$ this system is shift variant.

Note: $S\{g(t)\}(t) = g(t)\cos(t+\phi)$ is the same system as above.