

Signal Processing and Linear Systems1

Lecture 10: Windowing

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The Problem

We often want to work with short segments of a function.

Naively, we could multiply our function by a rect of a desired width.

This works, but the Fourier Transform of a rect is a sinc. The sinc function dies out slowly with lots of oscillations.

So the spectrum of the signal gets convolved with a sinc, which causes ringing.

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The Problem

Or we want to work with a subset of our spectrum.

Naively, we could multiply our spectrum by a rect of a desired width.

This causes ringing in the reconstructed signal.

We'd like a new way to keep just a part of our function that doesn't cause ringing in the other domain.

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Solution

$$f_w = f w \quad \Leftrightarrow \quad F_w = F * W$$

Multiply by a function w such that

- The portion of f that you care about isn't distorted much
- w is localized
- convolution with W doesn't cause ringing

w is called a window.

It can be used as a verb. We window the function.

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Spectral Leakage

For a given window width D and desired bandwidth B , we would like to minimize

$$\frac{\int_{|s|>B} |W(s)|^2 dx}{\int_{-\infty}^{\infty} |W(s)|^2 dx} .$$

That is, we would like the energy in the Fourier domain outside of our desired bandwidth (normalized by the total energy) to be minimized.

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The function that optimizes the previous metric is the Prolate Spheroidal Wave function.

However, this function is computationally expensive. There are some very good faster alternatives.

Jackson, John I., et al. "Selection of a convolution function for Fourier inversion using gridding (computerised tomography application)." IEEE transactions on medical imaging 10.3 (1991): 473-478.

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Two-Term Cosine

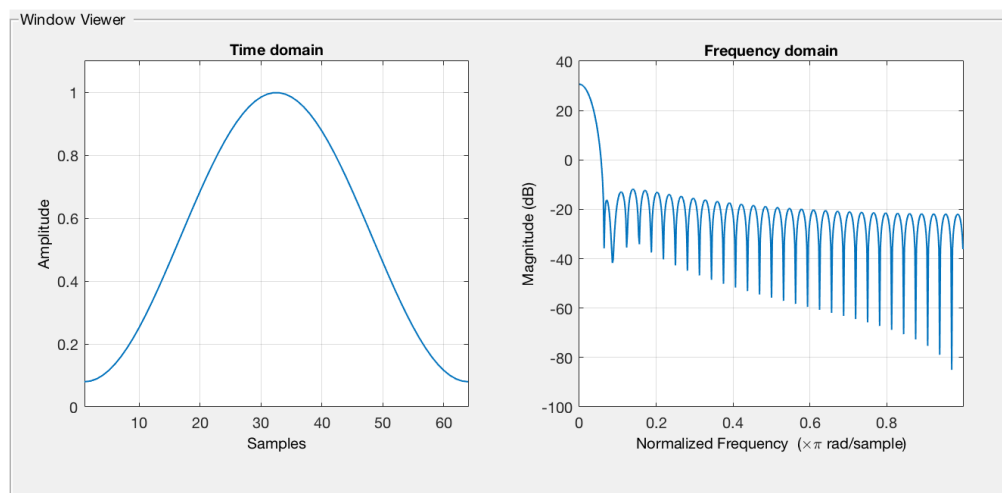
$$w(x) = \alpha + (1 - \alpha) \cos\left(\frac{2\pi}{D}x\right)$$

$w(x) = 0$ for all $|x| > D/2$.

Hamming Window: $\alpha = 0.54$

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Hamming Window



$$w(x) = \alpha + (1 - \alpha) \cos\left(\frac{2\pi}{D}x\right) \quad w(x) = 0 \quad \text{for all } |x| > D/2.$$

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Kaiser-Bessel Window

$$w_{KB}(x) = \frac{1}{D} I_0 \left(\beta \sqrt{1 - (2x/D)^2} \right)$$

D - the width of the window

I_0 - zeroth order modified Bessel function of the first kind

β - is a parameter of the window.

$w_{KB}(x) = 0$ for all $|x| > D/2$.

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Kaiser-Bessel Window

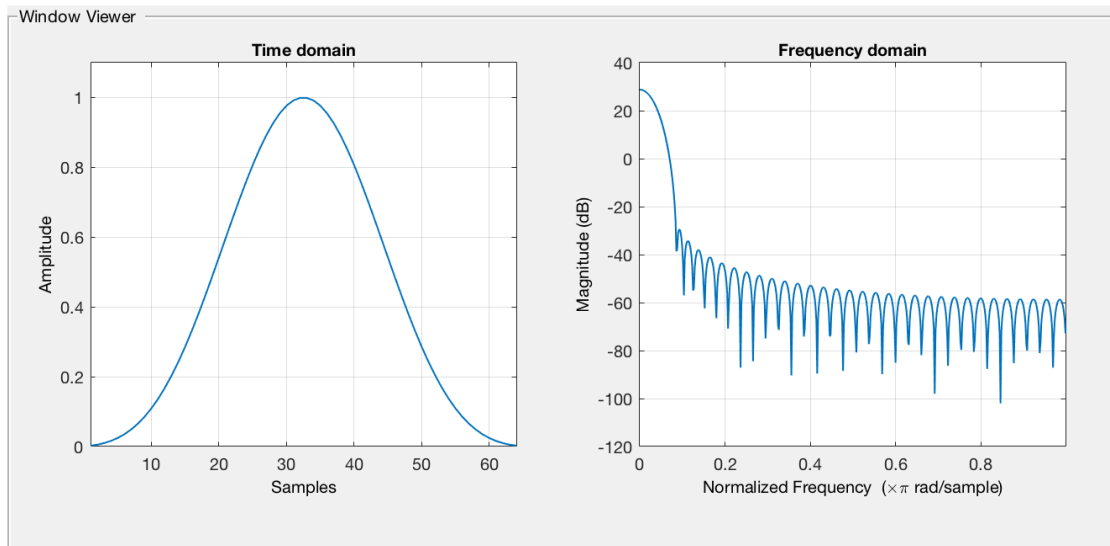
$$w_{KB}(x) = \frac{1}{D} I_0 \left(\beta \sqrt{1 - (2x/D)^2} \right)$$

$$W_{KB}(s) = \frac{\sin \sqrt{\pi^2 D^2 s^2 - \beta^2}}{\sqrt{\pi^2 D^2 s^2 - \beta^2}}$$

$w_{KB}(x) = 0$ for all $|x| > D/2$.

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Kaiser-Bessel Window



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The Kaiser-Bessel window offers a great trade-off between computational complexity and spectral leakage.

Jackson, John I., et al. "Selection of a convolution function for Fourier inversion using gridding (computerised tomography application)." IEEE transactions on medical imaging 10.3 (1991): 473-478.

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Pre-Emphasis

Note that each window alters the spectrum in the pass-band.

We can sometimes account for this distortion by altering the signal prior to windowing.

Multiply the spectrum of the signal by the inverse of the window in the pass-band (e.g. prior to transmission).

Then, after the windowing function, the signal is un-distorted in the pass-band.