Assignment 8 - Solutions

1/ Since $DFT\{x\}$ is imaginary and odd, x is real and odd. $\Rightarrow x[n] = -x[-n]$ for all n. When $n=0$, $x[0] = -x[0] \Leftrightarrow x[0] = 0$.

2/ $FS\{f_{p.e.}\}[n] = \frac{1}{L}\int_{-L/2}^{L/2} f(x)\, e^{-i2\pi \frac{nx}{L}}\, dx$.

Let $\hat{f}(x) = \begin{cases} f(x) & \text{for all } x \in [-L/2, L/2]. \\ 0 & \text{otherwise} \end{cases}$

$f_{p.e.}(x) = \hat{f}(x) * Ш_L(x)$ where $Ш_L(x) = \sum_{n=-\infty}^{\infty} \delta(x-nL)$.

Recall: $\mathcal{F}\{Ш_L\} = \frac{1}{L} Ш_{1/L}$.

$$\mathcal{F}\{f_{p.e.}\}(k) = \mathcal{F}\{\hat{f}(x) * Ш_L(x)\}(k) = \mathcal{F}\{\hat{f}\}(k) \cdot \mathcal{F}\{Ш_L(x)\}(k). \quad \cdots\cdots ①$$

$$\mathcal{F}\{\hat{f}\}(k) = \int_{-\infty}^{\infty} \hat{f}(x)\, e^{-i2\pi kx}\, dx = \int_{-L/2}^{L/2} f(x)\, e^{-i2\pi kx}\, dx.$$

$$\Rightarrow FS\{f_{p.e.}\}[n] = \frac{1}{L}\mathcal{F}\{\hat{f}\}(n/L).$$

From ①, $\mathcal{F}\{f_{p.e.}\}(k) = \mathcal{F}\{\hat{f}\}(k) \cdot \frac{1}{L} Ш_{1/L}$. Thus, the scaling ~~x~~ factors of L cancel out.

$\therefore$ $\mathcal{F}\{f_{p.e.}\}$ is a weighted delta function train where each weight is the corresponding value of the Fourier Series coefficients.

4/ The key to this problem is to look at the spectrum for very short periods of time.

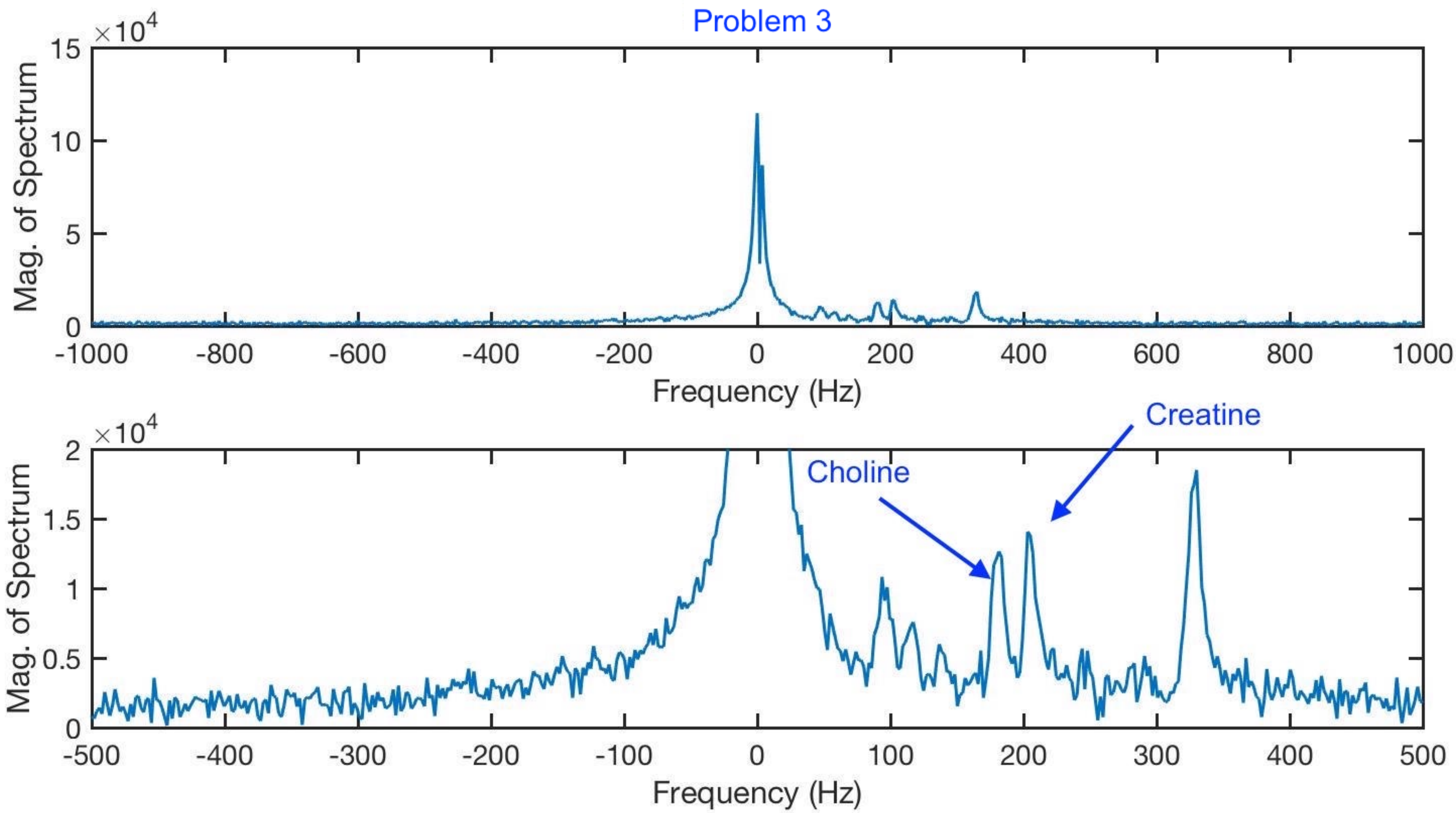
When you do that, you'll find that the keys pressed are

2 - 1 - 7 - 3 - 3 - 3 - 0 - 7 - 1 - 6.

5/ a) $y(t) = \frac{1}{2}\left(s_1(t) + i\, s_2(t)\right)$.

b) $s_1(t) = 2\,\mathrm{Re}\{y(t)\}$. $\quad s_2(t) = 2\,\mathrm{Im}\{y(t)\}$.

c) The signals can not be recovered.



Problem 3
×10^4
15
10
5
0
Mag. of Spectrum
-1000
-800
-600
-400
-200
0
200
400
600
800
1000
Frequency (Hz)
×10^4
2
1.5
1
0.5
0
Mag. of Spectrum
-500
-400
-300
-200
-100
0
100
200
300
400
500
Frequency (Hz)
Choline
Creatine