EE 102A - Assignment 6

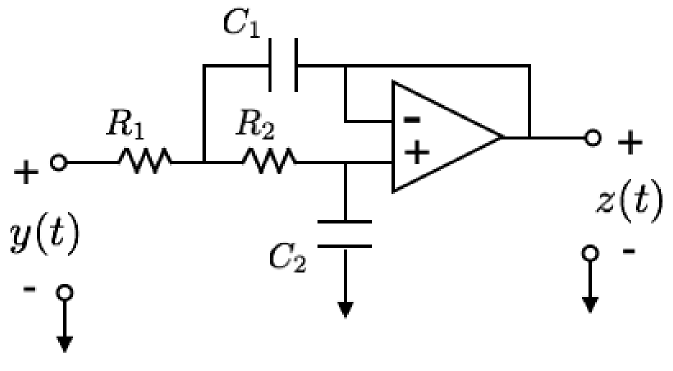
Nicholas Dwork

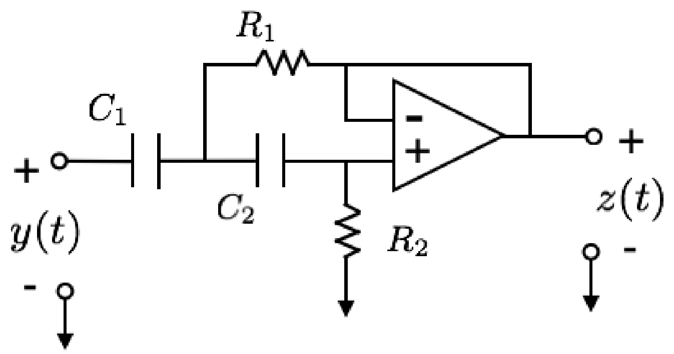
Problem 1. Consider $a, b \in \mathbb{C}^N$. Show DFT$\{a \circledast b\} =$ diag(DFT$\{a\}$) DFT$\{b\}$.

Problem 2. *DFT Representations*
For this problem, all vectors have 64 elements. That is, $x = (0, 1, 2, \ldots, 63)$.
a) Plot the Power Spectral Density (PSD) of $\cos(2\pi\, 9/64\, x)$.
b) Plot the Power Spectral Density (PSD) of $\cos(2\pi\, 10/64\, x)$.
c) Plot the Power Spectral Density (PSD) of $\cos(2\pi\, 9.5/64\, x)$.
d) What can you infer from what just happened?
Recall that the PSD of a vector x equals $|\text{DFT}\{x\}|^2$.

Problem 3. *By Moosa Zaidi*
In class, we showed that convolution in the space domain corresponds to multiplication in the frequency domain: $\mathcal{F}\{f * g\} = \mathcal{F}\{f\}\, \mathcal{F}\{g\}$. In this problem, you will show that multiplication in the space domain corresponds to convolution in the frequency domain: $\mathcal{F}\{f\, g\} = \mathcal{F}\{f\} * \mathcal{F}\{g\}$.
a) Show that $\mathcal{F}^{-1}\{f * g\} = \mathcal{F}^{-1}\{f\}\, \mathcal{F}^{-1}\{g\}$.
b) Using the result from part (a), show that $\mathcal{F}\{f\, g\} = \mathcal{F}\{f\} * \mathcal{F}\{g\}$.

Problem 4. *Butterworth Filters*
Consider the following two circuits.





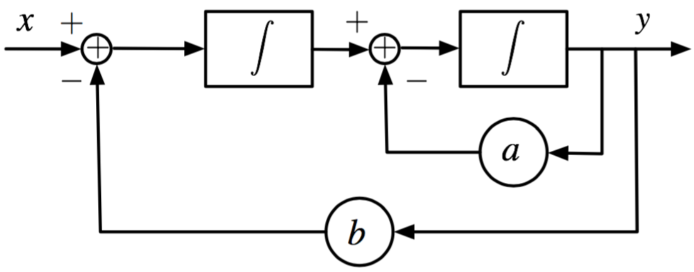
a) Find the transfer function of both circuits. One is a high pass filter and one is a low pass filter. Which one is which?
b) Combine the two filters (in some way) to create a bandpass filter. What is the transfer function of your new filter?
c) Choose circuit elements so that the filter suppresses frequencies below 100 kHz and above 150 kHz. Plot the magnitude of the transfer function of your bandpass filter.

Problem 5. Consider a system with input x and output y governed by the following ordinary differential equation:

$$\sum_{m=0}^{M-1} a_m \, y^{(m)} = \sum_{n=0}^{N-1} b_n \, x^{(n)},$$

where $y^{(m)}$ is the m^{th} derivative of the function y. Show that this system is linear and shift invariant.

Problem 6. *System with Feedback*
Find the differential equation that governs the system shown below.



Problem 7. *10 points*
a) Determine whether the following systems are linear or non-linear, shift invariant or shift variant, causal or non-causal, and have memory or are memoryless. Justify all your answers.

$$S\{y(\gamma)\}(t) = \int_{-\infty}^{t} y(\gamma)d\gamma$$

$$S\{y(\gamma+3)\}(t) = y(t+2) - y(t-1)$$