EE 102A - Assignment 5

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Problem 1. *Properties of Sinc*
a) Compute the following:

$$\int_{-\infty}^{\infty} |\text{sinc}(x)|^2 dx.$$

b) Find the area under the curve of a sinc function.

Problem 2. *Non-Fourier Transformable*
If a function f meets all of the following conditions (called the Dirichlet conditions) then the synthesis of the Fourier Transform equals the original function.

- The function f must be absolutely integrable. That is, $\int_{-\infty}^{\infty} |f(t)|\, dt < \infty$.
- The function f has a finite number of maxima and minima within any finite interval.
- The function f must have a finite number of discontinuities within any finite interval. Furthermore, each of these discontinuities must be finite.

a) Provide an example of a bounded differentiable function that does not meet the Dirichlet conditions.
b) Provide an example of a bounded differentiable function that does not meet the third Dirichlet conditions.

Problem 3. *Proof of the Convolution Theorem*
a) Show that $\nu(x) = \exp(i\, 2\pi f x)$ is an eigenfunction of any linear shift-invariant system. What is its corresponding eigenvalue?
b) Use this to prove the Convolution Theorem.

Problem 4. *Audio Filtering*
At the following link, you can download a segment of *Burn it Down* by Linkin Park ('cause they totally rock!!!).

```
https://www.stanford.edu/~ndwork/teaching/1706ee102a/hmwk/data/burnItDown.wav
```

An audio segment is just a 1D array. You can read this data into Matlab and listen to it using the following commands.

```
[bid, Fs] = audioread( 'burnItDown.wav' );
soundsc( a, Fs );
```

Give it a shot and make sure that you can hear the music.
One can approximate the Fourier Transform with a computer using the Discrete Fourier Transform. The command in Matlab to do that is

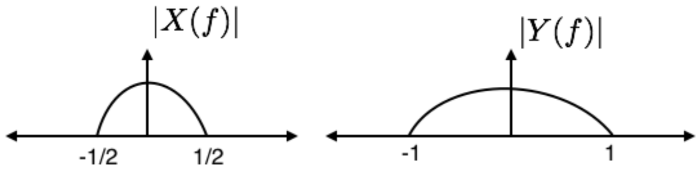
```
bidSpectrum = fftshift( fft( bid ) );
```

The complex array `bidSpectrum` is now the spectrum of the audio signal. The 0 frequency component is in the middle of the array, the highest frequencies (in magnitude) are near the beginning and the end of the array.
One can reconstruct the signal from its spectrum using the inverse Discrete Fourier Transform as follows.

```
bidRecon = ifft( ifftshift( bidSpectrum ) );
```

a) Read the data into Matlab and plot the sound signal.
b) Apply three separate filters to the sound signal: a low-pass filter, a band-pass filter, and a high-pass filter. Plot the power spectral density functions of the resulting filtered signals.
c) Listen to each of the filtered signals. What do you hear?

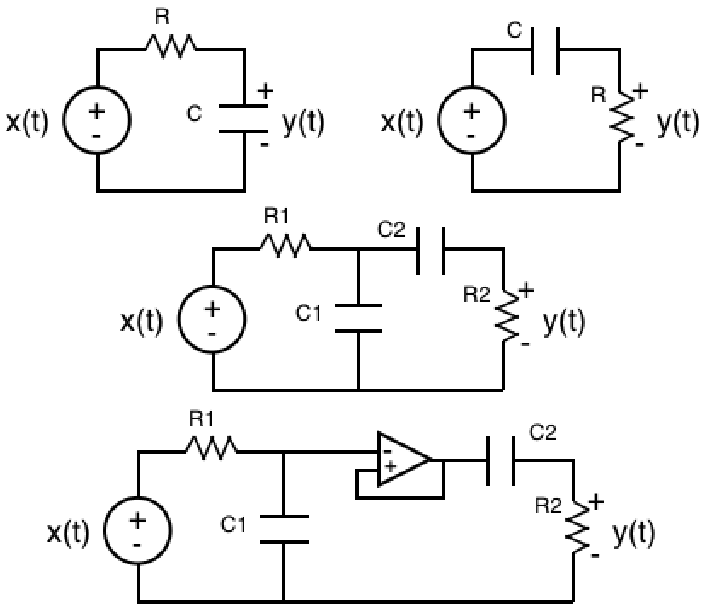
Problem 5. The magnitudes of the spectrums of two signals x and y are shown in the figure below.



Find the bandwidths of each of the following functions.
a) $x * y$
b) xy
c) $f(t) = \int_{-\infty}^{t} x(\tau) d\tau$
d) $g(\gamma) = x(\gamma)\, \cos(2\pi\gamma)$
e) $f(\nu) = x(\nu)\, \exp{(i2\pi\nu)}$
f) $g = (x * y)'$

Problem 6. *First-Order Difference*
The first order difference is a way to estimate the derivative of a sampled signal. It converts a vector $x = (x_1, x_2, \ldots, x_N)$ with N samples into a new vector $y = (x_2 - x_1, x_3 - x_2, \ldots, x_N - x_{N-1})$.
a) Consider the system $S\{x\} = y$. Show that this system is linear. Is this system shift invariant?
b) Find a matrix D such that $y = Dx$.

Problem 7. *Bandpass filter*
Find the transfer functions of the following circuits. (Note: an op-amp in this configuration is called a *voltage follower*.)



a) How does the transfer function of the fourth circuit relate to the transfer function of the first two? What does this tell you about voltage followers?
b) Are there any conditions under which the voltage follower is not needed? That is, are there any conditions under which the transfer function of the third circuit is approximately equal to the product of the transfer functions of the first two.
c) Choose resistor and capacitor values to create a bandpass filter that permits frequencies between 100 kHz and 150 kHz.
d) Plot the magnitude of the transfer function of your bandpass filter.
e) Plot the magnitude of the spectrum of the output if $x(t) = \Pi(t - 0.5)$.

Problem 8. *Alternate Fourier Transform*
The definition of the Fourier Transform that we've been using in this class is

$$F(s) = \mathcal{F}\{f\}(s) = \int_{-\infty}^{\infty} f(x)\,\exp\left(-i2\pi sx\right)\,dx.$$

Another popular definition of the Fourier Transform is

$$\hat{F}(\omega) = \hat{\mathcal{F}}\{f\}(\omega) = \int_{-\infty}^{\infty} f(x)\,\exp\left(i\omega x\right)\,dx.$$

(Note the sign of the argument of the exponential.)
Derive a formula for the inverse of $\hat{\mathcal{F}}$.

Problem 9. *Generalized Parseval's Theorem*
Consider two Fourier transformable functions f and g and their Fourier Transforms F and G. Prove the following.

$$\int_{-\infty}^{\infty} f(x)\,\overline{g}(x)\,dx = \int_{-\infty}^{\infty} F(k)\,\overline{G}(k)\,dk$$