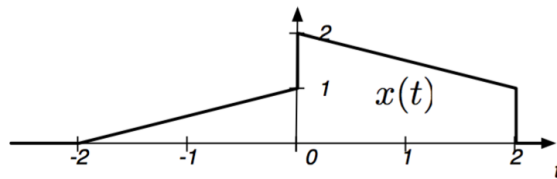


EE 102A - Assignment 3

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Please justify all your answers.

Problem 1. Consider the following function x .



Draw the following signals:

- $x\left(2\left(-t + \frac{1}{2}\right)\right)$
- $x\left(\frac{t-1}{2}\right)$
- $x(2(1-t))$
- $x\left(\frac{t}{2} - 1\right)$
- Graph the even and odd components of this function.

Problem 2. Prove the following properties of the Dirac delta function:

- $\int_{-\infty}^{\infty} x(s) \delta(s - \Delta) ds = x(\Delta)$
- $\delta(as) = 1/|a| \delta(s)$

Problem 3. Prove the following properties of convolution:

- $f * (g * h) = (f * g) * h$
- $f * g = g * f$
- $f * (g + h) = (f * g) + (f * h)$
- $f * \delta = f$

Problem 4. For each of the following, find a function h such that $y = x * h$.

- $y(t) = \int_{-\infty}^t x(\tau) d\tau$
- $y(s) = \int_{s-T}^s x(\tau) d\tau$
- $y(\gamma) = x(\gamma)$
- $y(\nu) = x(\nu - 1)$
- $y(\nu) = x(\nu + 1)$

Problem 5. Computing Convolutions

- Compute $f * g$ where $f(x) = u(x)$ and $g(x) = \Pi(x - 1)$ and plot the result.
- Compute $\Pi * \Pi$ and plot the result. What is the name of this function?

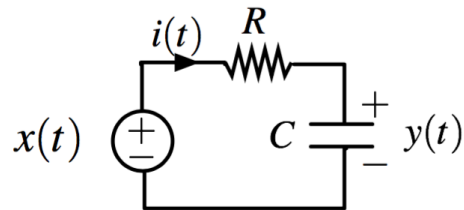
Problem 6. Integrals and Derivatives

- Compute $f(x) = \int_{-\infty}^x \delta(\gamma) d\gamma$. What is the name of the function f ?
- Find the derivative of Π . (Use the fact that the derivative is the inverse operation of integration.)

Problem 7. Derivatives of the Dirac Delta

- a) Find the value of $\int_{-\infty}^{\infty} \delta'(s) f(s) ds$.
- b) Find the value of $\int_{-\infty}^{\infty} \delta^{(k)}(s) f(s) ds$ where $\delta^{(k)}$ is the k^{th} derivative of δ .

Problem 8. Consider the RC circuit shown in the figure below:



When we consider x (the voltage of the source as a function of time) to be the input and y (the voltage across the capacitor as a function of time) to be the output, then this circuit can be modeled as a system. The voltage source is turned on at time $t = 0$; prior to that, the circuit is at rest (the voltage across the capacitor is 0 and there is no current going through the circuit).

- a) Is this system linear? Is the system shift invariant?
- b) Find the impulse response of this system.
- c) Find the output of this system when $x(t) = \Pi(t - 0.5)$.
- d) Find the output of this system when $x(t) = \cos(t)u(t)$ (where u is the Heaviside step function).

Problem 9. Image Steganography

This project will show you a way to hide a message in an image.

Each pixel of an image is a number that ranges from 0 to 255. This is because we store the data in a pixel as a byte, which is an ordered set of eight bits (a bit is a single digit that can store either 0 or 1). The conversion from binary to decimal is as follows:

$$x_7 x_6 x_5 x_4 x_3 x_2 x_1 x_0 = 2^7 x_7 + 2^6 x_6 + \dots + 2^0 x_0.$$

For example:

$$10110010 = 2^7 + 2^5 + 2^4 + 2^1 = 178.$$

The quantity x_0 is called the *Least Significant Bit* or LSB. Note that the gray of 178 is extremely close to the gray of 179. So if we added or subtracted 1 to the value it wouldn't change the image much.

To encode an ASCII message into an image, subtract 1 from all odd values. What is the LSB after we do this? Then, add the message bit (1 or 0) to each pixel in sequence until the message is completed. Do not add anything to the remaining pixels.

To convert from ASCII to binary (and vice-versa) you can use the following website:

<http://www.unit-conversion.info/texttools/convert-text-to-binary/#data>

Extract the message in the image located at

www.stanford.edu/~ndwork/teaching/si2016/session2/hmwk3/bacEmbedded.png.

In this image, the ASCII code was embedded in the image in column-major order.