Midterm Solutions

1 a) $S_1\{y(\gamma)\}(t) = \int_{-\infty}^{t} y(\gamma)\,d\gamma.$

$S_2\{y(\gamma+3)\}(t) = y(t+2) - y(t-1).$

S_1 is linear, and causal. It has memory.

$$S_1\{y(\gamma-\Delta)\}(t) = \int_{-\infty}^{t} y(\gamma-\Delta)\,d\gamma = \int_{-\infty}^{t-\Delta} y(u)\,du.$$

$\therefore S_1$ is shift invariant.

To make S_2 more clear, let $g(\gamma) = y(\gamma+3)$.

Then $S_2\{y(\gamma+3)\}(t) = S_2\{g(\gamma)\}(t)$

$= g(t-1) - g(t-4).$

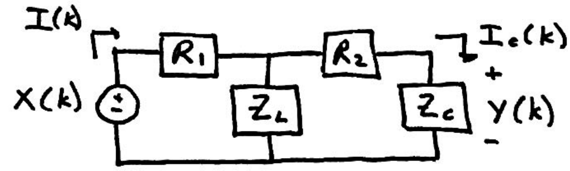
S_2 is linear, shift invariant, causal, and has memory.

b) The impulse response $h = S\{\delta\}$.

a) $h(t) = u(t)$, where u is the Heaviside step function.

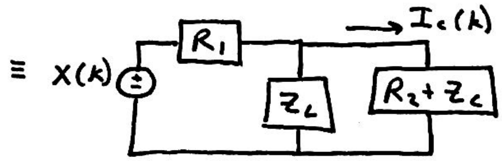
b) $h(t) = \delta(t-1) - \delta(t-4).$

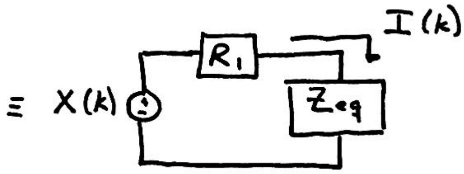
2) We start by drawing a schematic of the circuit in the Fourier domain.



$$Z_C(k) = \frac{1}{j2\pi Ck}$$

$$Z_L(k) = j2\pi Lk.$$





$$Z_{eq}(k) = \left(\frac{1}{Z_L} + \frac{1}{R_2+Z_C}\right)^{-1} = \frac{Z_L(R_2+Z_C)}{R_2+Z_C+Z_L}.$$

$$I(k) = \frac{X(k)}{R_1+Z_{eq}(k)} = X(k)\left(\frac{R_2+Z_C+Z_L}{R_1(R_2+Z_C+Z_L)+Z_L(R_2+Z_C)}\right).$$

$$I_C(k) = I(k)\left(\frac{Z_L}{R_2+Z_C+Z_L}\right)$$

$$= X(k)\left(\frac{Z_L}{R_1(R_2+Z_C+Z_L)+Z_L(R_2+Z_C)}\right).$$

$$Y(k) = I_C(k)\,Z_C(k) = X(k)\left(\frac{Z_L Z_C}{R_1(R_2+Z_C+Z_L)+Z_L(R_2+Z_C)}\right).$$

$$\therefore\ H(k) = \frac{Y(k)}{X(k)} = \frac{Z_L Z_C}{R_1(R_2+Z_C+Z_L)+Z_L(R_2+Z_C)}.$$

3) Since the phase is 0 for all f, H is a real function. I.e. $H = |H|$.

Let $y(x)$ denote the output.

$Y(f) = H(f)\, G(f)$. $\quad G(f) = \Pi(f) + \frac{1}{2}\Pi(f/2)$.

$\Rightarrow Y(f) = \Pi(f/2)$. $\quad \therefore\ y(x) = 2\,\mathrm{sinc}(2x)$.

4) See homework or notes.